

**MATHEMATICS
HIGHER LEVEL
PAPER 1**

Candidate number

--	--	--	--	--	--	--	--

Tuesday 4 November 2003 (afternoon)

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet
e.g. Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

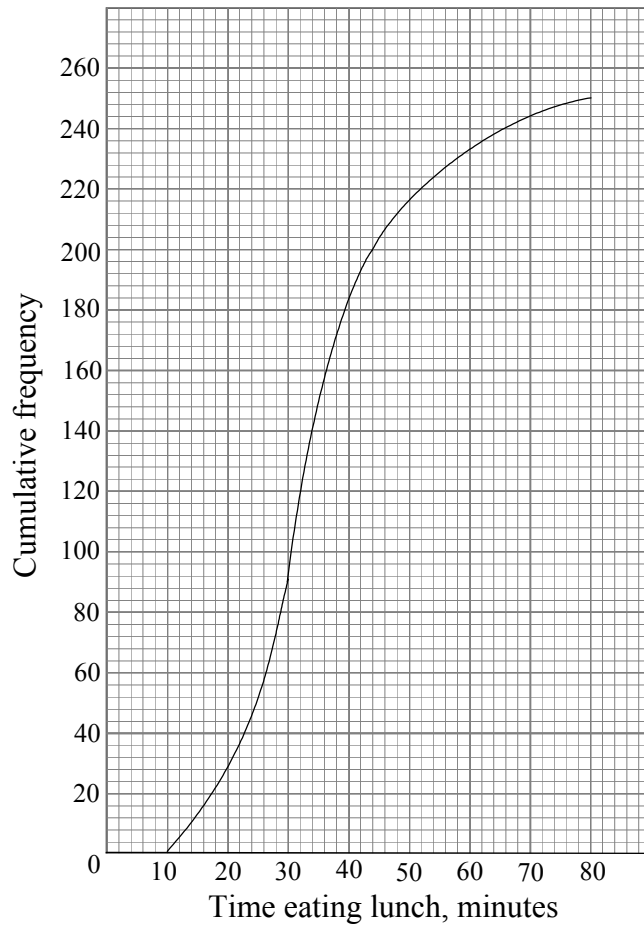
1. Consider the points $A(1, 2, -4)$, $B(1, 5, 0)$ and $C(6, 5, -12)$. Find the area of $\triangle ABC$.

Working:

Answer:

2. The cumulative frequency curve below indicates the amount of time 250 students spend eating lunch.

- (a) Estimate the number of students who spend between 20 and 40 minutes eating lunch.
- (b) If 20 % of the students spend more than x minutes eating lunch, estimate the value of x .



Working:

Answers:

(a) _____

(b) _____

3. The matrices A , B , C and X are all non-singular 3×3 matrices.

Given that $A^{-1}XB = C$, express X in terms of the other matrices.

Working:

Answer:

4. A continuous random variable, X , has probability density function

$$f(x) = \sin x, \quad 0 \leq x \leq \frac{\pi}{2}.$$

Find the median of X .

Working:

Answer:

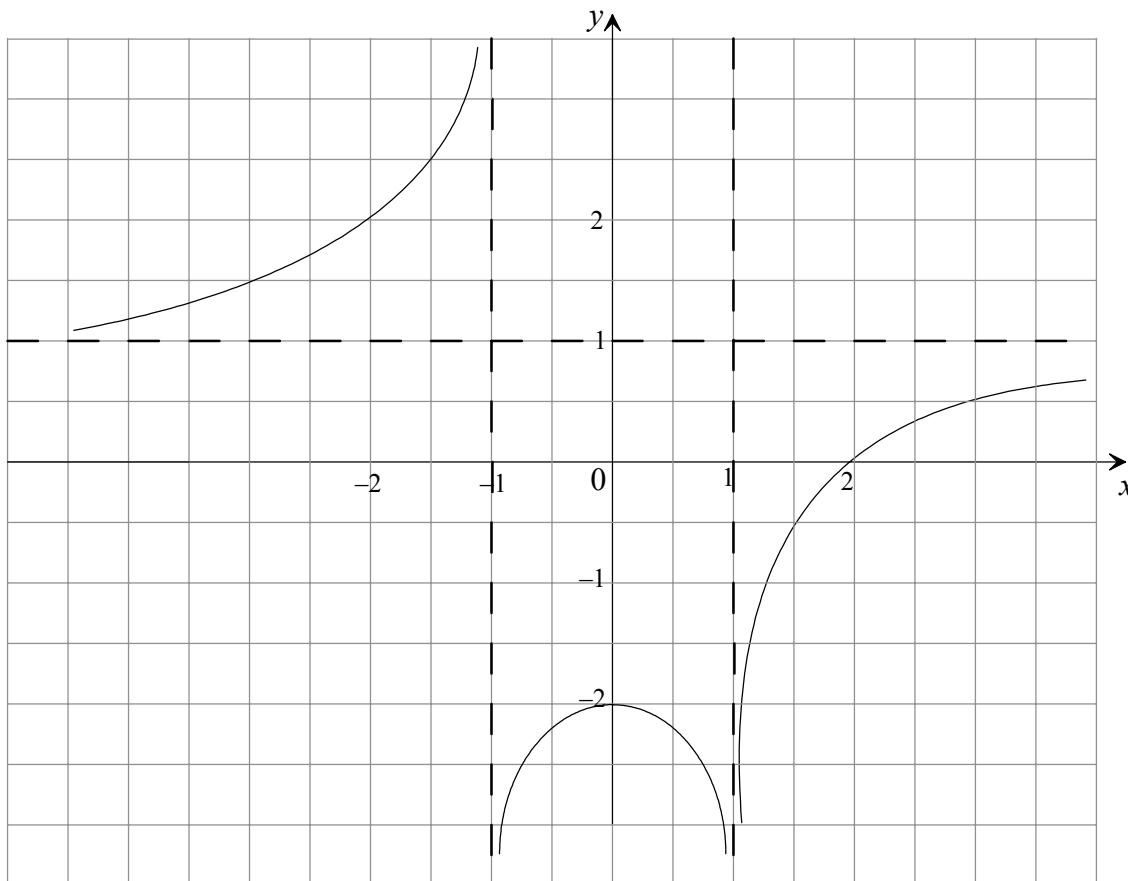
5. Consider the equation $2(p + iq) = q - ip - 2(1 - i)$, where p and q are both real numbers. Find p and q .

Working:

Answers:

6. The diagram shows the graph of $f(x)$.

(a) On the same diagram, sketch the graph of $\frac{1}{f(x)}$, indicating clearly any asymptotes.



(b) On the diagram write down the coordinates of the local maximum point, the local minimum point, the x -intercepts and the y -intercept of $\frac{1}{f(x)}$.

Working:

7. Find the angle between the plane $3x - 2y + 4z = 12$ and the z -axis. Give your answer to the nearest degree.

Working:

Answer:

8. Consider the function $f(t) = 3 \sec^2 t + 5t$.

(a) Find $f'(t)$.

(b) Find the **exact** values of

(i) $f(\pi)$;

(ii) $f'(\pi)$.

Working:

Answers:

(a) _____

(b)(i) _____

(ii) _____

9. The first four terms of an arithmetic sequence are 2 , $a - b$, $2a + b + 7$ and $a - 3b$, where a and b are constants. Find a and b .

Working:

Answers:

10. Solve $\log_{16} \sqrt[3]{100-x^2} = \frac{1}{2}$.

Working:

Answers:

11. Calculate the area enclosed by the curves $y = \ln x$ and $y = e^x - e$, $x > 0$.

Working:

Answer:

12. On a television channel the news is shown at the same time each day. The probability that Alice watches the news on a given day is 0.4 . Calculate the probability that on five consecutive days, she watches the news on at most three days.

Working:

Answer:

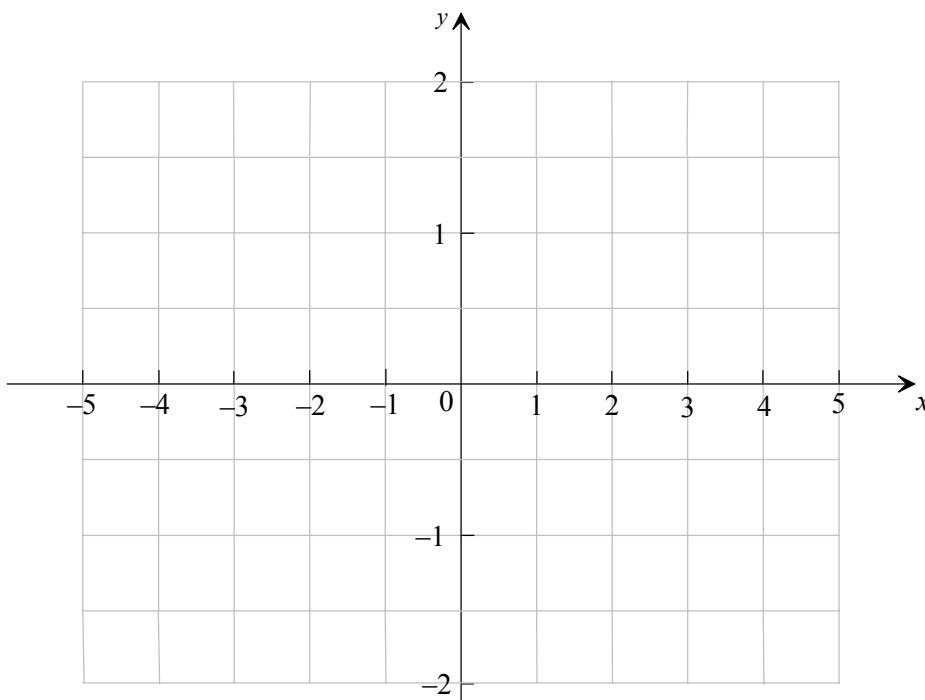
13. Consider the equation $(1 + 2k)x^2 - 10x + k - 2 = 0$, $k \in \mathbb{R}$. Find the set of values of k for which the equation has real roots.

Working:

Answer:

14. Let $f(x) = \sin\left(\arcsin\frac{x}{4} - \arccos\frac{3}{5}\right)$, for $-4 \leq x \leq 4$.

(a) On the grid below, sketch the graph of $f(x)$.



(b) On the sketch, clearly indicate the coordinates of the x -intercept, the y -intercept, the minimum point and the endpoints of the curve of $f(x)$.

(c) Solve $f(x) = -\frac{1}{2}$.

Working:

Answer:

(c) _____

15. Consider the equation $2xy^2 = x^2y + 3$.

(a) Find y when $x = 1$ and $y < 0$.

(b) Find $\frac{dy}{dx}$ when $x = 1$ and $y < 0$.

Working:

Answers:

(a) _____

(b) _____

16. Let $y = e^{3x} \sin(\pi x)$.

(a) Find $\frac{dy}{dx}$.

(b) Find the smallest positive value of x for which $\frac{dy}{dx} = 0$.

Working:

Answers:

(a) _____

(b) _____

17. Let $f(x) = \frac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \frac{x-2}{x-4}$, $x \neq 4$.

Find the set of values of x such that $f(x) \leq g(x)$.

Working:

Answer:

18. A committee of four children is chosen from eight children. The two oldest children cannot both be chosen. Find the number of ways the committee may be chosen.

Working:

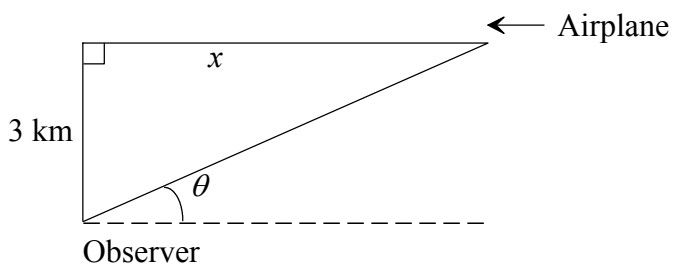
Answer:

19. Solve $2(5^{x+1}) = 1 + \frac{3}{5^x}$, giving the answer in the form $a + \log_5 b$, where $a, b \in \mathbb{Z}$.

Working:

Answer:

20. An airplane is flying at a constant speed at a constant altitude of 3 km in a straight line that will take it directly over an observer at ground level. At a given instant the observer notes that the angle θ is $\frac{1}{3}\pi$ radians and is increasing at $\frac{1}{60}$ radians per second. Find the speed, in kilometres per hour, at which the airplane is moving towards the observer.



Working:

Answer: